I- (20 pts) Find a matrix X such that $AXB = C$ given that:

$$A=\begin{bmatrix} 1 & -2 \\ -2 & 3 \\ 1 & 4 \end{bmatrix},\ B=\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \text{ and } C=\begin{bmatrix} 8 & 6 & -6 \\ 6 & -1 & 1 \\ -4 & 0 & 0 \end{bmatrix}$$

II- (20 pts) Solve the following system using Gauss-Jordan elimination:

$$\begin{cases} 2x_1 - 3x_2 = -2x_3 - 5x_4 + 3 \\ x_1 - x_2 = -x_3 - 2x_4 + 1 \\ 3x_1 + 2x_2 = -2x_3 - x_4 \\ x_1 + x_2 = 3x_3 + x_4 \end{cases}$$

III- (20 pts) Find the matrix A using the given information:

$$(I+2A)^{-1}=\begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}$$

IV- (20 pts) **<u>1)</u>** Let A be a symmetric matrix ($A^T = A$):

a) Show that A^2 is symmetric
b) Show that $2A^2-3A+I$ is symmetric
c) If A is also invertible, show that A^{-1} is symmetric

<u>2)</u> Let A be is skew-symmetric matrix ($A^T = -A$):

d) Show that if A is an invertible skew-symmetric matrix, then A^{-1} is skew-symmetric.

V- (20 pts) Determine the values of "a" so that the following system of equations has: i) no solution, ii) more than one solution, iii) a unique solution:

$$\begin{cases} x + y - z = 1 \\ 2x + 3y + az = 3 \\ x + ay + 3z = 2 \end{cases}$$